Revenue maximization in unlimited-supply setting

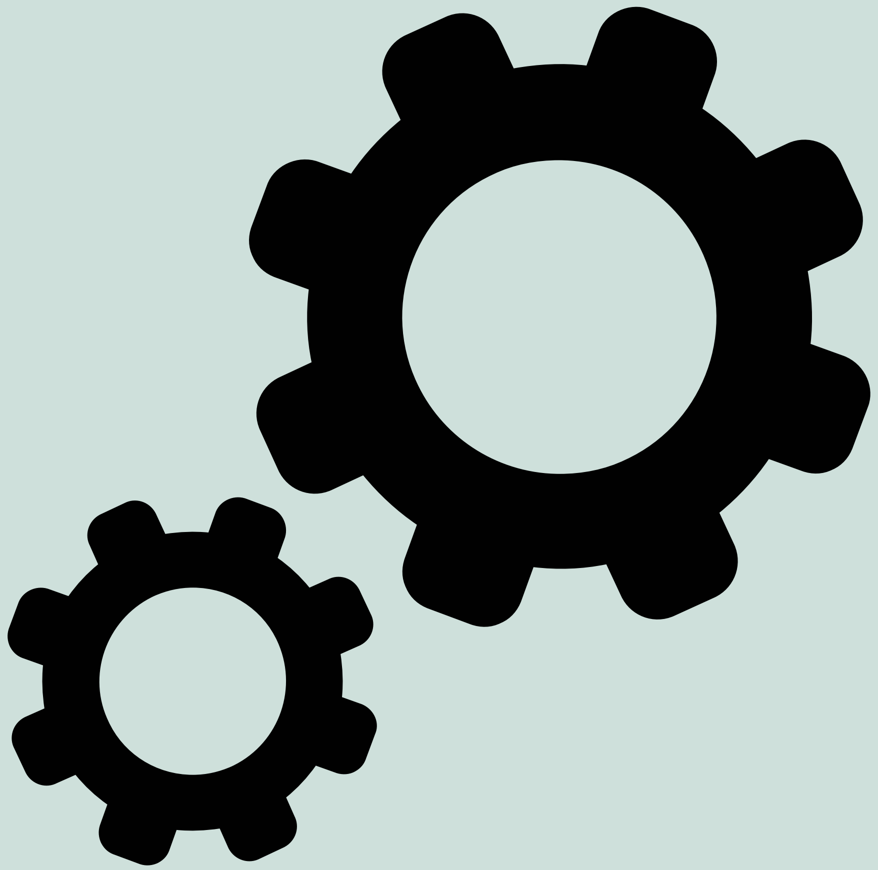
Prayus Shrestha | CPSC 532L

Agenda

(1) Setup
(2) Intuition & Problem
(3) Possible Resolution
(4) Next Steps

Setup

What is the setting we're working with?



Setup

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  - Unlimited supply: It costs the seller ~$0 to produce more units of that good

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  - Unlimited supply: It costs the seller ~$0 to produce more units of that good
- n = |N| buyers for the good, each with some valuation $v_i \in [1, V]$
  - Buyers buy the good if the price is less than or equal to their own valuation
  - Valuations are private, but seller can have some overall info

Intuition & Problem

How does the game play out? What's wrong with this?

Seller's Goal

Maximize {Profit = Revenue - Cost}

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Maximize {Profit = Revenue - Cost}

Maximize {Profit = Revenue - 0}

Seller's Goal

Maximize {Profit = Revenue - Cost}

Maximize {Profit = Revenue - 0}

Maximize {Revenue = Price * (# of Buyers)}

Revenue Maximization

Maximize {Revenue}, given v = [v1, v2, ..., vn]:

- Sort valuations in descending order
  - Buyer with the i-th highest valuation will be in position i
- $i^* = \operatorname*{argmax}_{i \in N} \{v_i \cdot i\}$
- Price = v_{i^*}; Revenue = $v_{i^*} \cdot i^*$

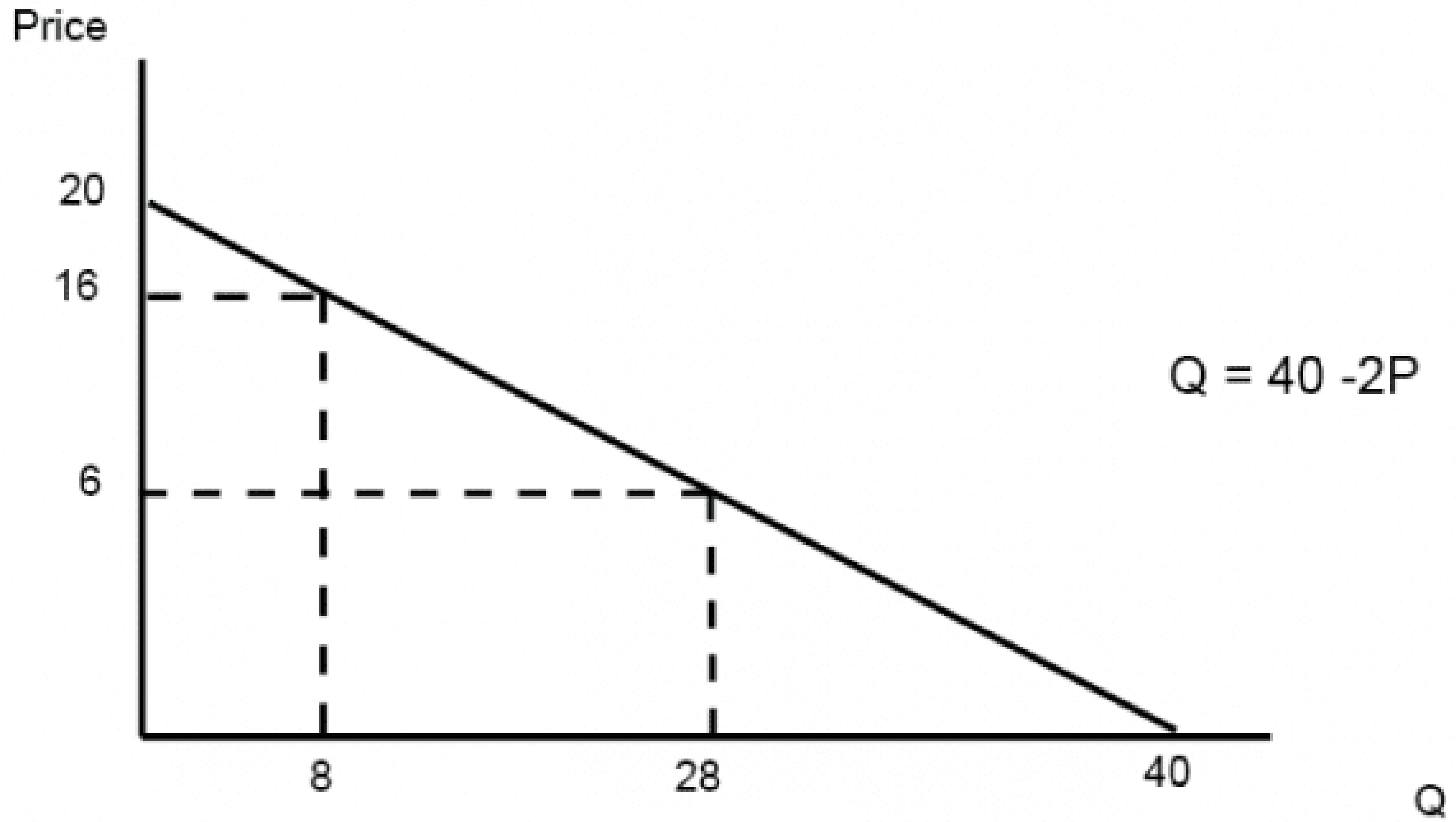
Revenue Maximization Example

Maximize {Revenue}, given one buyer, with their valuation being drawn from v = [V, V/2, ..., V/V]:

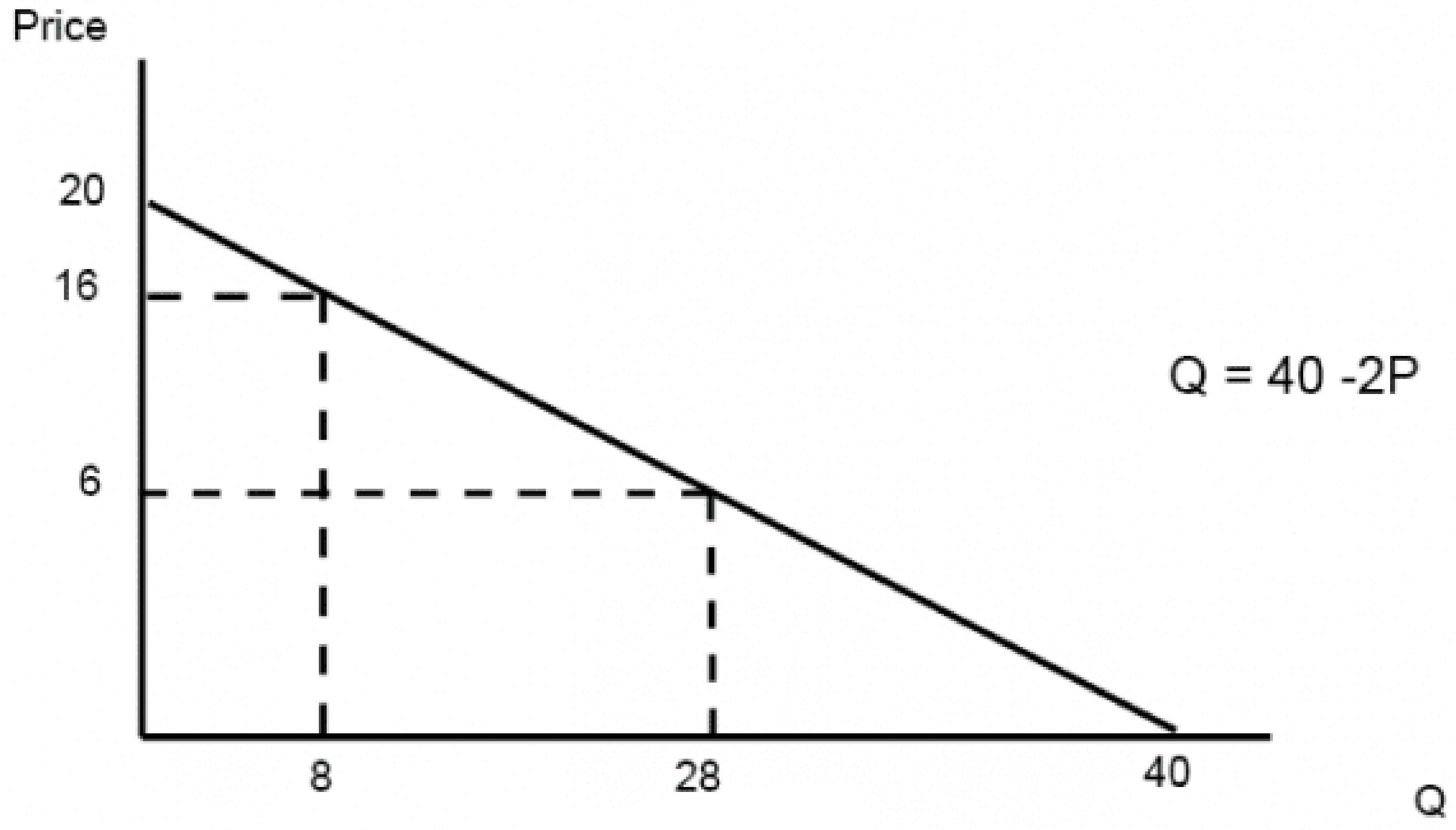
Set price $= \frac{V}{i}, i \in \{1, 2 \ldots, V\} \rightarrow$ probability of the buyer having a valuation $\geq \frac{V}{i}$ is $\frac{i}{V}$

Revenue is: price $\times$ # of buyers $= \frac{V}{i} \cdot \frac{i}{V} = 1$

Social Welfare vs. Revenue



Social Welfare vs. Revenue



Social welfare maximized when price = 0

- Social welfare = 1/2 * 40 * 20 = 400
- Here, the social welfare is captured by just the buyers and the seller gets nothing

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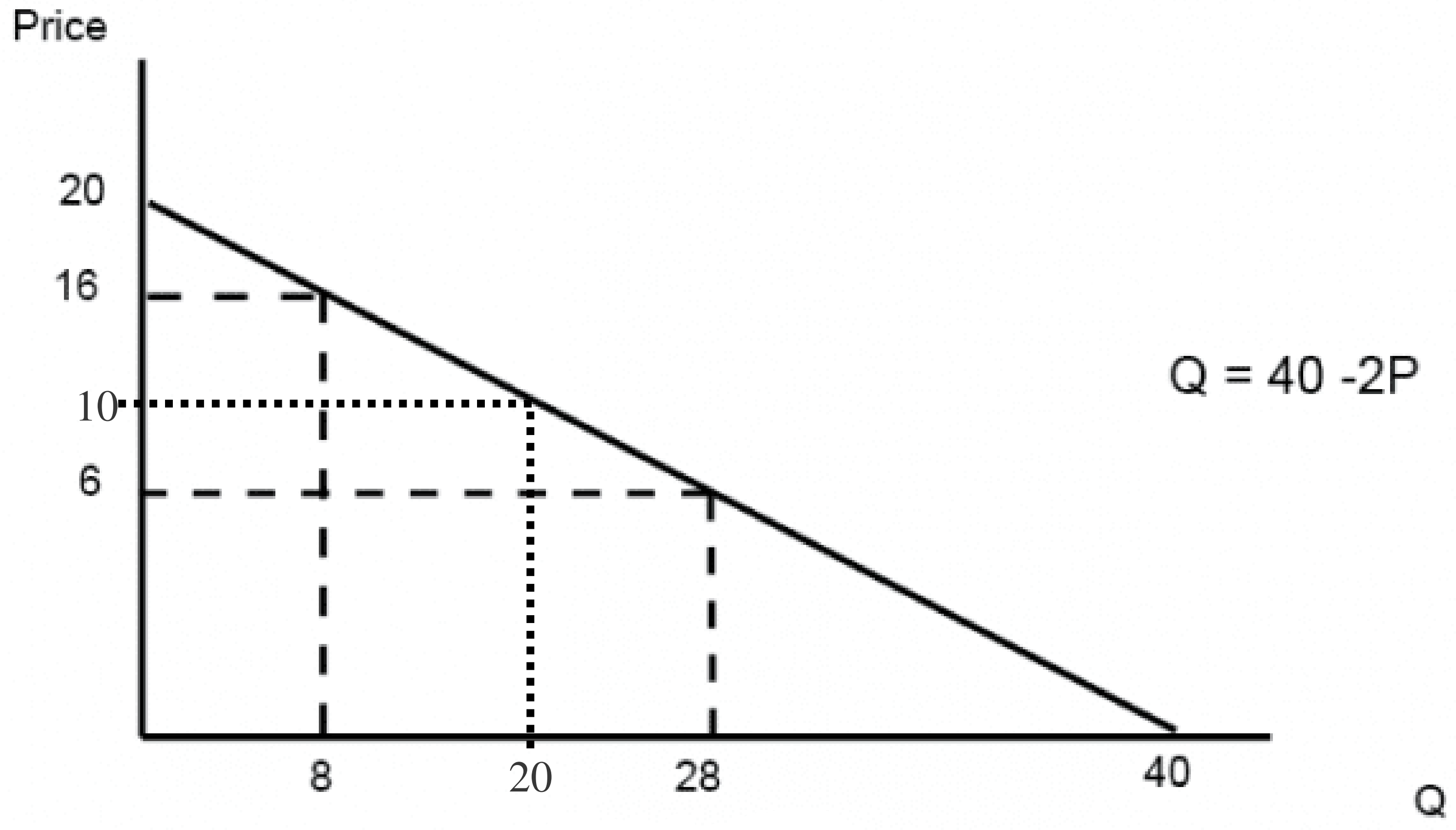

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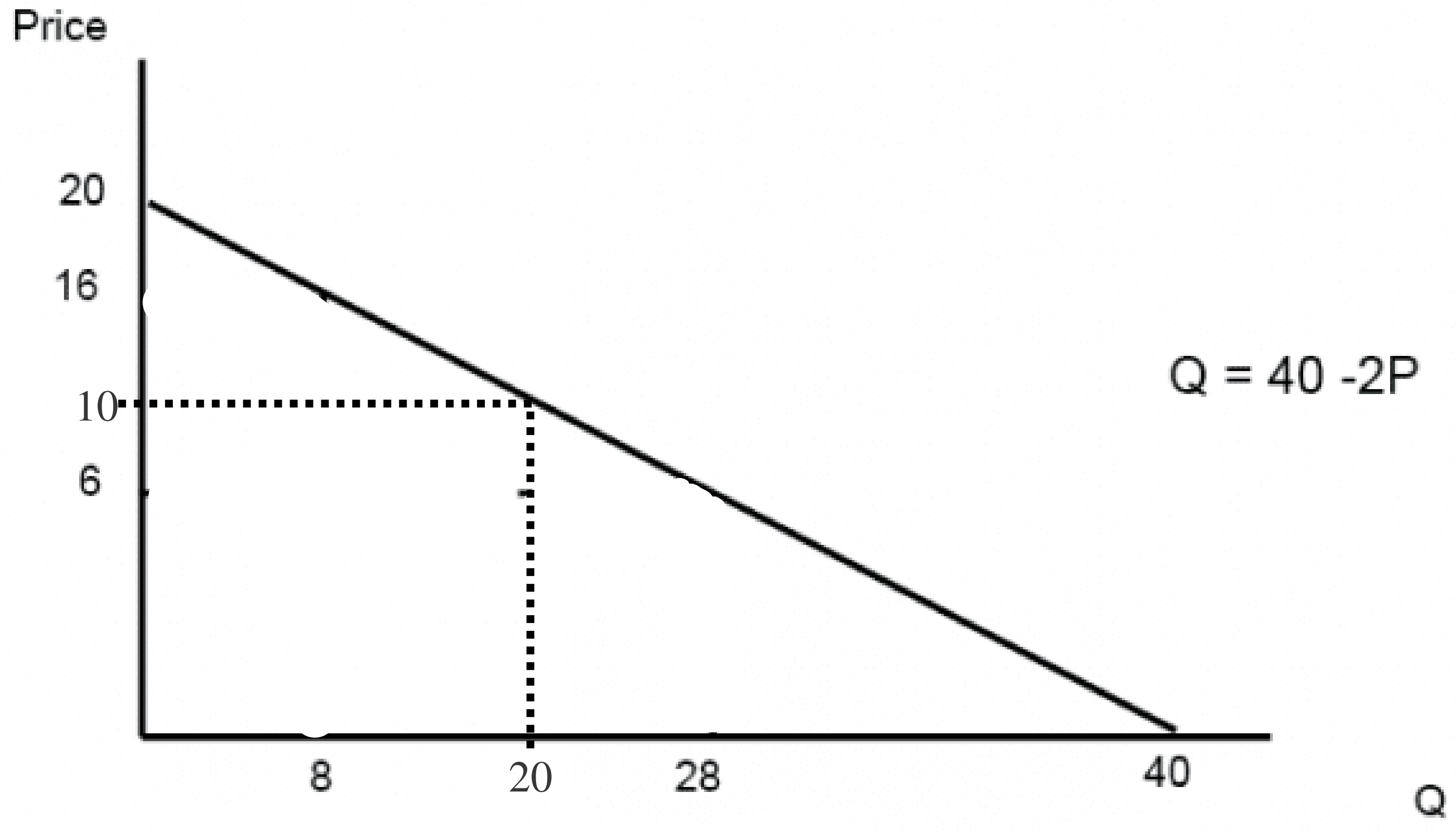
- Revenue = P * (40 - 2P) maximized when p=10
  - Revenue = 10 * 20 = 200

Social Welfare vs. Revenue



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- Revenue = P * (40 - 2P) maximized when p=10
  - Revenue = 10 * 20 = 200
- Social welfare = 200 + (1/2 * 20 * 10) = 300
- Here, the lower social welfare is mostly captured by the seller

Possible Comprimise

Can we find some middle ground?

Possible Resolution

Profit Extractors

- Let R be the profit target that is less than or equal to the maximum revenue
  - Profit Extractors aim to give the seller that profit and/or revenue

- Profit Extractors find the largest number of buyers (k) such that each buyer values the good at least R/k; once this k is found, these buyers (with the top k valuations) each purchase the good for price R/k

Profit Extractors

```
function profit_extractor(R: int, v: Valuations, N: Buyers) -> k: int:
    /**
     * R: target revenue
     * v: buyer valuations in descending order
     * N: set of buyers
     *
     * k: max number of buyers st. each buyer values the good at least R/k
     */

    for k in [|N|, |N|-1, ..., 1]:
        price = R/k
        if price > v[k]:
            remove that buyer from N
        else:
            return k
    return -1
```

Possible Resolution

Beyond this Presentation

Combinatorial Auction

- Michael's presentation next week!

Twists to this Setting

- What if agents were negatively impacted if they didn't buy the good but their "competitor" did?

Finding R

- Setting an R without knowing the distribution of valuations:
  - Random Sampling Auction
  - Consensus Estimates

Next steps

References

Competitive Auctions, Goldberg et al. (2006)

Algorithmic Game Theory, Roughgarden, Tardos, Vazirni (2007)

Single Price Mechanism for Revenue Maximization in Unlimited Supply Combinatorial Auctions, Balcan, Blum, Mansour (2007)

Multiunit Auctions, Leyton-Brown (2008)

Auction of Digital Goods with Externalities, Rui (2018)